Stellar Atmospheres: Structure, Composition and Limb Darkening

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References

- **Some general texts**

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Introductory remarks:

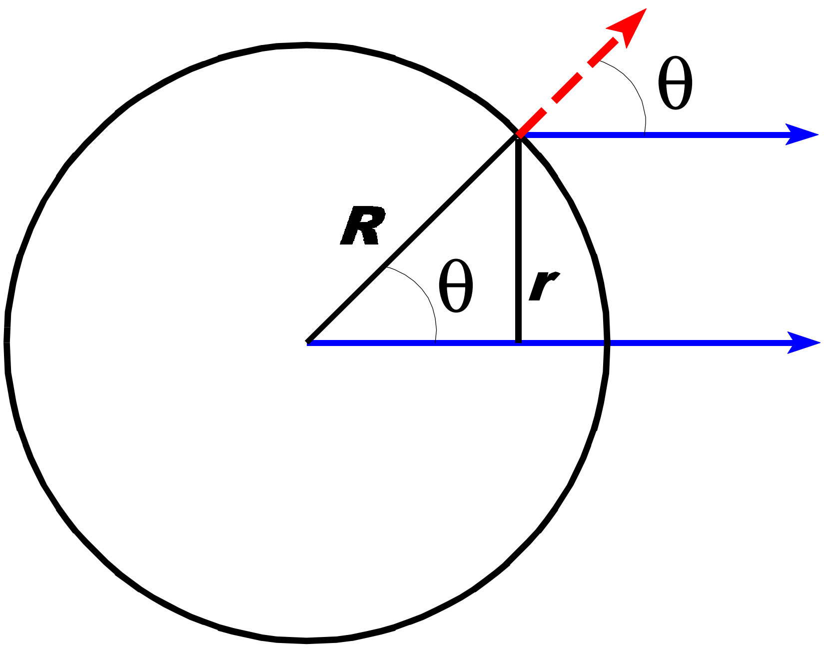
- **Resolved stars (*e.g*. the Sun) are observed to be darker at the edge (*i.e*. limb) than at the center of the disk.**

- **The phenomenon arises because stars have a temperature gradient — they are hotter in the deeper parts than in the outer parts of the atmosphere.**

- **By studying limb darkening at a variety of wavelengths we may hope to understand something about the atmospheric chemical composition and structure.**

- **Limb darkening studies, once restricted to very few stars, now broadly possible with the advent of optical interferometers.**

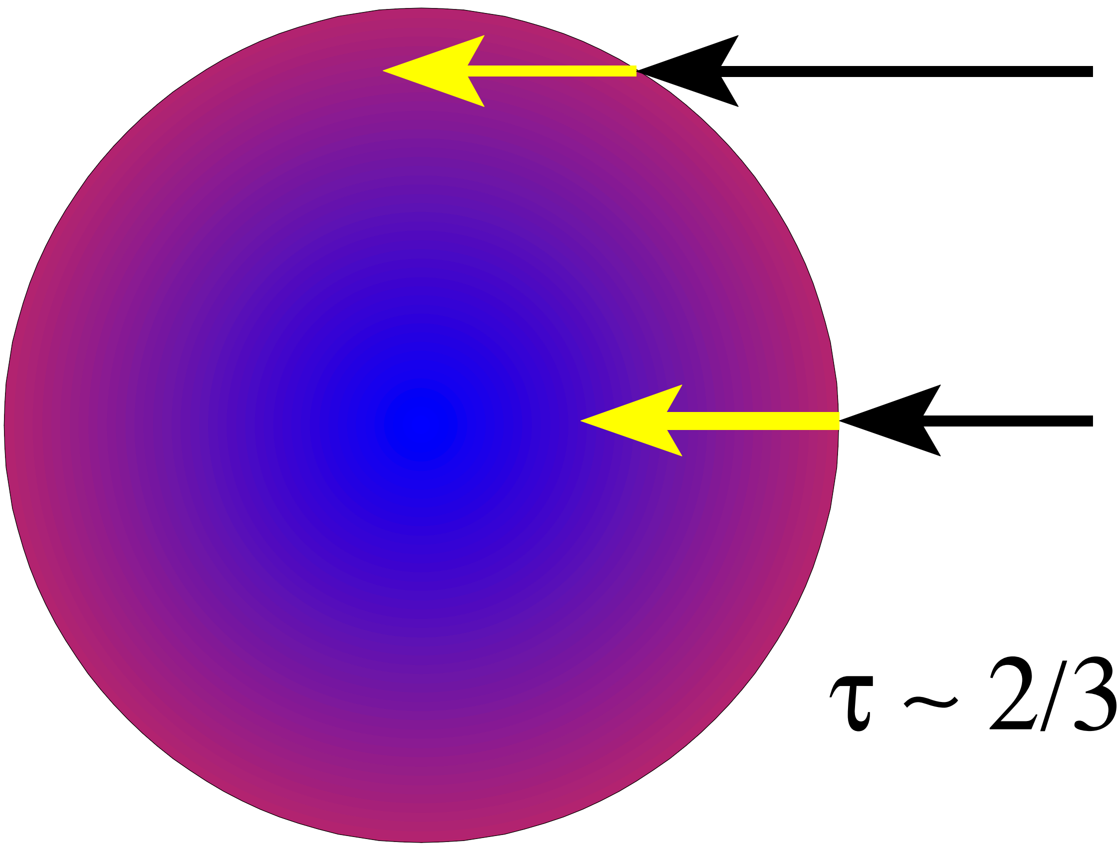
Limb-darkening geometry:



Definitions:

- *R* = radius of the star.
- *r* = distance projected along disk from center.
- θ = angle between the normal to the stellar surface and the line of sight to the observer.
- μ = cos θ.

Simplified picture of limb darkening:

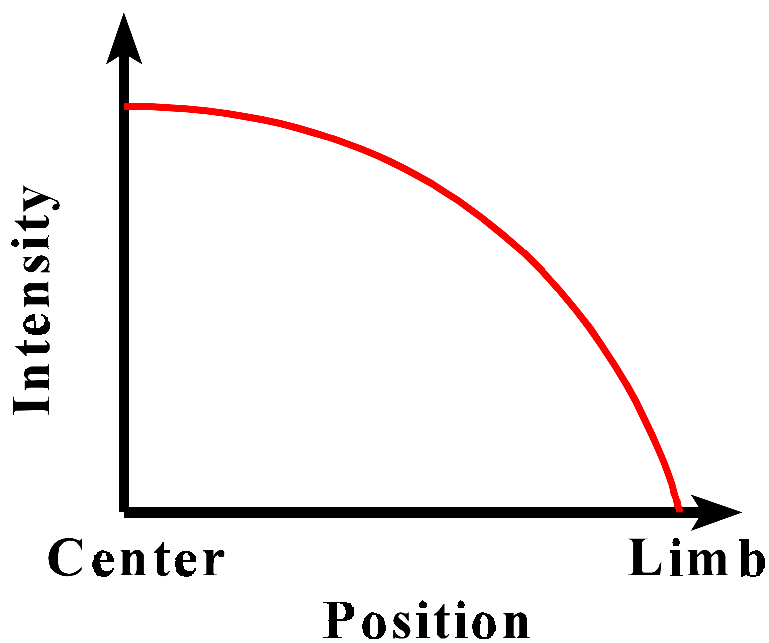


Our expectations:

- **The observed intensity is a function of T, τ_λ:**

$$I_\lambda(\mu) = f\,[B_\lambda(T), \tau_\lambda]$$

- **Expect lower intensities at limb than at center:**



- **Observe $I_\lambda(\mu)$ and infer T, τ_λ with the aid of radiative transfer models**

Radiative transfer formalism:

- **The emergent intensity in the plane-parallel approximation is**

$$I_\lambda(\mu) = \int_0^\infty S_\lambda(\tau_\lambda)\, e^{-\tau_\lambda/\mu}\, d\tau_\lambda/\mu$$

where $S_\lambda(\tau_\lambda)$ is the *source function*.

- **$S_\lambda(\tau_\lambda)$, τ_λ carry information about chemical composition, temperature and pressure in the stellar atmosphere.**

How to make some progress:

- **The relationship between intensity and source function is a Laplace transform:**

$$\mu I_\lambda(\mu) = \mathcal{L}[S_\lambda(\tau_\lambda)]$$

- **Observe I_λ (μ), find the inverse Laplace transform and obtain $S_\lambda(\tau_\lambda)$.**

- **Assume LTE and obtain $T(\tau_\lambda)$ because $S_\lambda(\tau_\lambda) = B_\lambda(\tau_\lambda)$, the *Planck function*.**

The Eddington approximation:

- Emergent intensity found to be

$$I(\mu) = I(1)\frac{3}{5}\left(\mu + \frac{2}{3}\right)$$

- Leads to the following source function

$$S(\tau) = \frac{3}{4\pi}F\left(\tau + \frac{2}{3}\right)$$

where F is the flux of radiation.

- Get relation between T and τ:

$$T^4 = \frac{3}{4}T_{eff}^4\left(\tau + \frac{2}{3}\right)$$

Limb-darkening laws

- The limb-darkening law in the Eddington approximation is a *linear limb-darkening law*. A more general form is

$$I(\mu) = I(1)(a + b\mu)$$

and may be found in the papers by van Hamme (1993) and Claret *et al*. (1995).

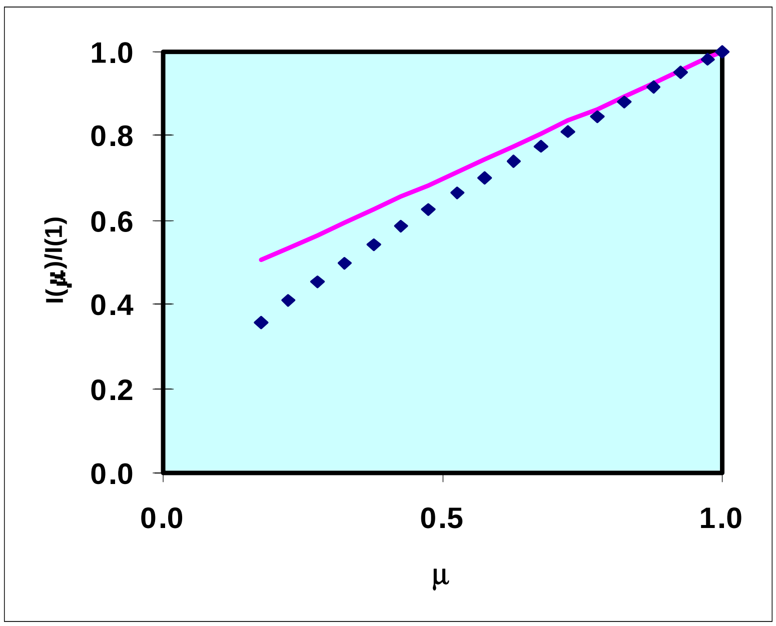
- Other polynomial forms have been suggested.

- A useful general limb-darkening function first suggested by Michelson & Pease (1921) and recently advocated by Hestroffer (1997) is

$$I(\mu) = I(1)\mu^{\alpha}$$

Solar limb-darkening:

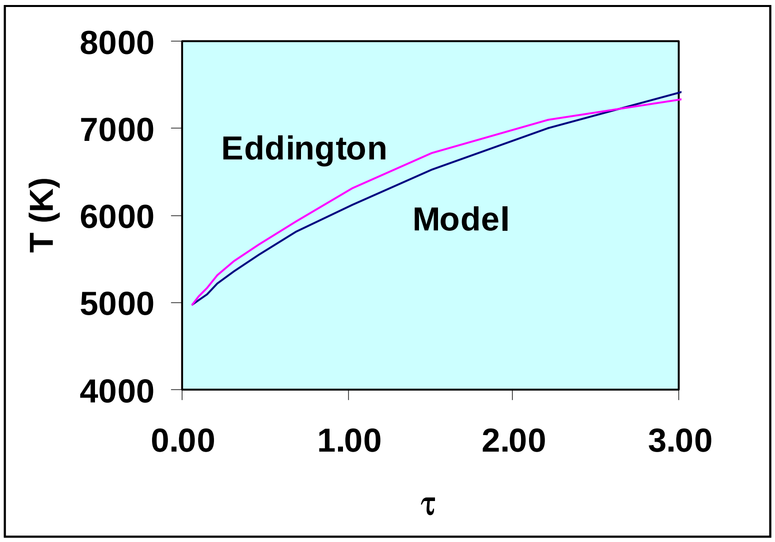
- Petro *et al.* (1984) obtained the following mean Solar limb-darkening at a wavelength of 445.1 nm. The Eddington approximation to the limb-darkening is shown for comparison.



- Eddington slope limb-darkening law is a poor fit. Petro *et al.* use a 5th degree polynomial to fit the intensity, yielding S_λ represented by a 5th degree polynomial in τ_λ.

Solar temperature distribution:

- **The following plot compares the *T*(τ) derived from the Eddington approximation for the Sun to detailed model from Kurucz (1979).**



- **Eddington approximation temperatures agree with model to ≈ 2% over the range 0.1 ≤ τ ≤ 3 even though limb-darkening fit is not good.**

- **Simple example illustrates how analysis of limb darkening leads to understanding of atmospheric temperature structure. Interpreting *real* observations is more complicated.**

Example inversion of real data:

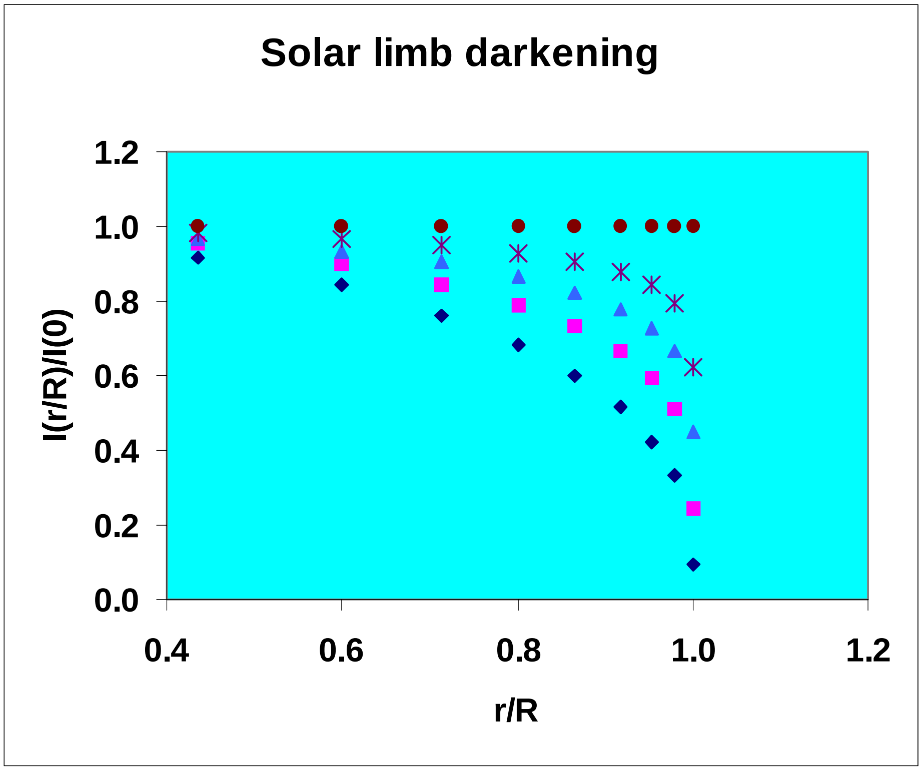
- **Solar limb-darkening data used by Pierce & Waddell (1961) to derive atmospheric temperature structure.**

- **Analysis of multi-wavelength data also shows how Solar continuous opacity derived from limb darkening observations.**

- **One of several investigations confirming H^- as an important constituent in the Solar atmosphere.**

- **Investigation shows how temperature *and* composition of the atmosphere may be determined from limb-darkening observations at a variety of wavelengths.**

Some observational data:



Notes:

- Horizontal row of dots at top are a uniformly-bright disk (*i.e.*, no limb darkening).
- Wavelengths for observed data are, starting from the lowest row of dots, 0.42, 0.6, 1.0 and 2.1 µm.

Some comments:

- **Sharp edge to the Sun, evident at all wavelengths.**
- **Wavelength dependence to limb darkening with near infrared behaving most like a uniform disk.**

What's next?

- **Assume that the source function may be written as the following expansion:**

$$\frac{S_\lambda(\tau_\lambda)}{I_\lambda(1)} = a_\lambda + b_\lambda \tau_\lambda + c_\lambda E_2(\tau_\lambda)$$

where $E_2(\tau_\lambda)$ is an exponential integral and a_λ, b_λ and c_λ are constants to be determined.

- **This results in a limb darkening function of the form:**

$$\frac{I_\lambda(\mu)}{I_\lambda(1)} = a_\lambda + b_\lambda \mu + c_\lambda [1 - \mu \ln(1 + \mu^{-1})]$$

- **Fit the observed limb darkening data and determine the value of the constants at each wavelength.**

- Assume LTE which gives

$$\frac{S_\lambda(\tau_\lambda)}{I_\lambda(1)} = \frac{B_\lambda(\tau_\lambda)}{I_\lambda(1)}$$

- Measure the absolute intensity at disk center, $I_\lambda(1)$ and numerically compute a solution.

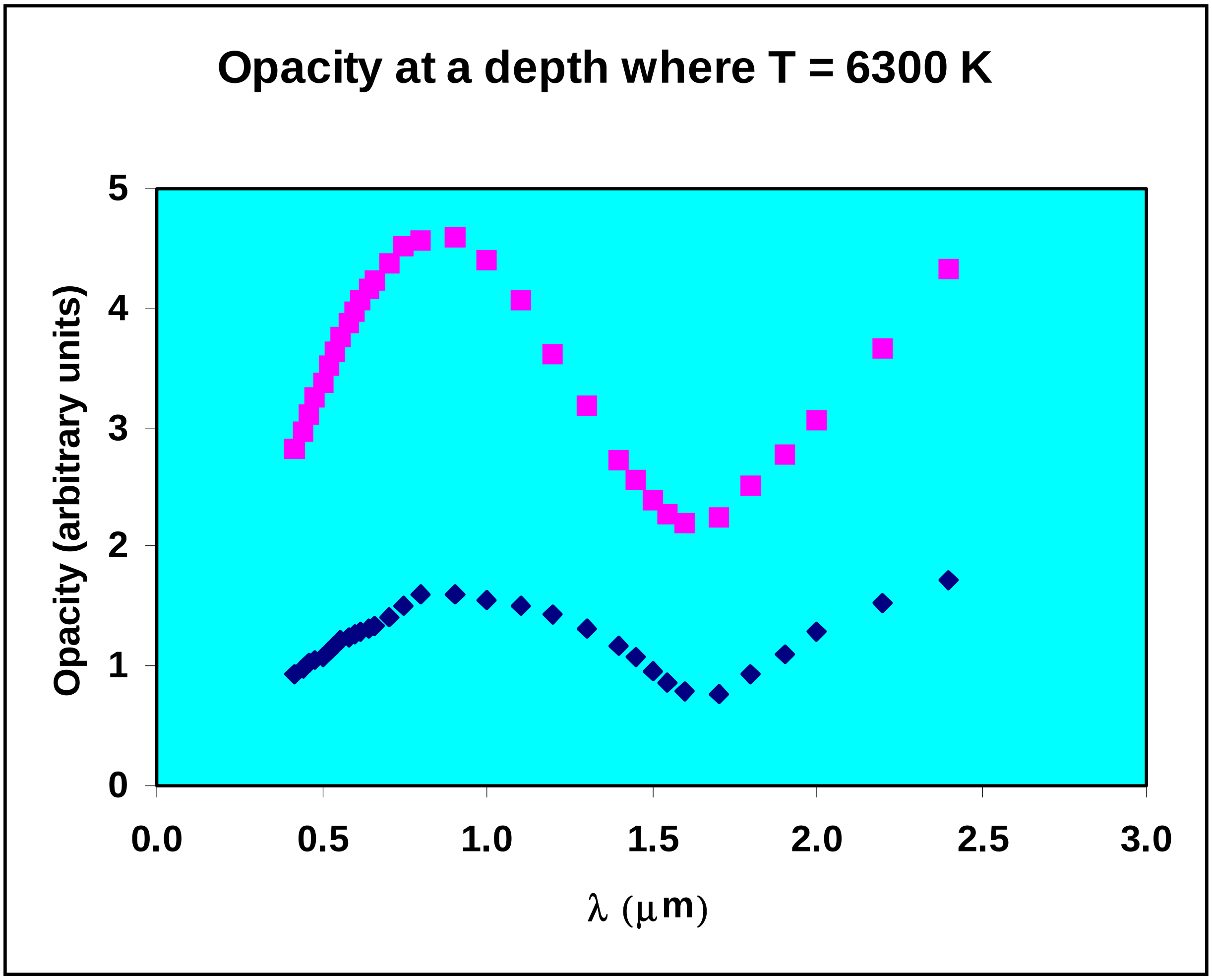
- Results in a table $T_\lambda(\tau_\lambda)$ for all wavelengths observed.

- For a fixed T_λ, τ_λ will vary because of the changing opacity since, by definition,

$$\tau_\lambda = \int_x^0 k_\lambda \rho \, dx$$

over some path length x, where k_λ is the absorption coefficient of the material and ρ is the density.

Results of the Pierce & Waddell study:



Bottom curve is shape of opacity derived from observations, upper curve is theoretical opacity for H^-.

Other stars:

- **Sun is special case since it is a very resolved disk.**

- **For other stars, interferometers must be used to resolve the disks.**

- **Interferometers measure the *visibility* rather than the center-to-limb variation (CLV) directly.**

- **Recall the Michelson & Pease CLV:**

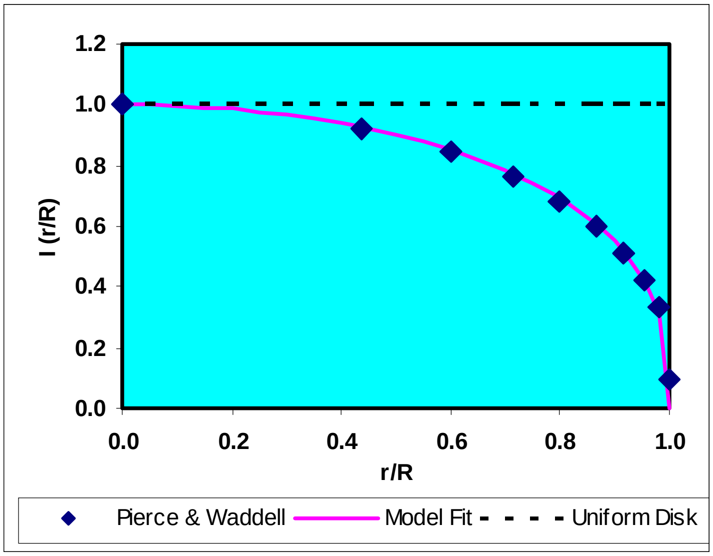
$$I(r/R) = I(0)[1-(r/R)^2]^{\alpha/2}$$

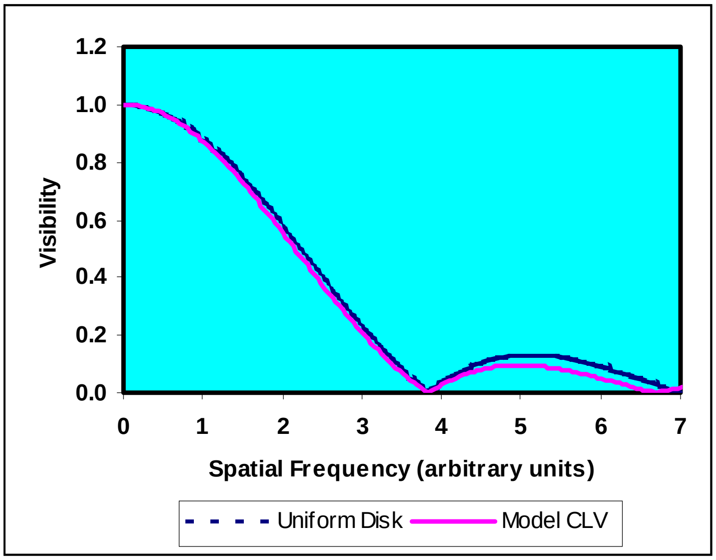
- **Hestroffer shows that this leads to the following visibility as a function of spatial frequency, *s*:**

$$V_\nu(s) = \Gamma(\nu+1)\,\frac{|J_\nu(s)|}{(s/2)^\nu}$$

where $\nu = (\alpha/2) + 1$, J_ν is the ν-th Bessel function of the first kind and Γ is the gamma function.

Application to the Sun:





Salient points of the previous graphs:

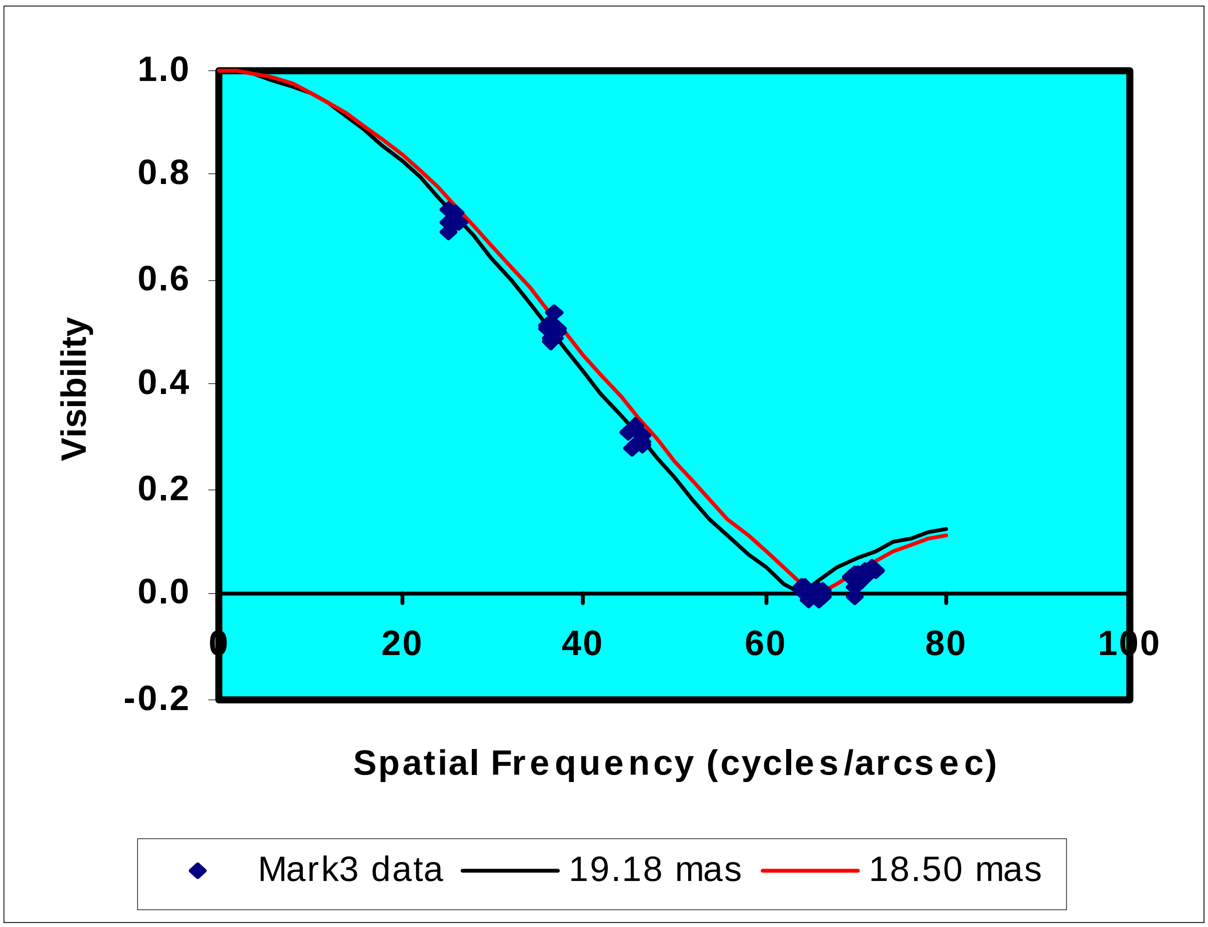
- **Michelson & Pease model fits CLV observations pretty well.**
- **Not much difference between uniform disk and limb-darkened visibilities where *s* is smaller than the first null.**
- **Limb-darkening information contained in height of secondary maxima and positions of nulls.**
- **Limb-darkening effect is small, so high accuracy measurements are needed.**

Summary for stars with T > 4000 K:

- **Atmosphere models pretty good.**
- **Observed limb-darkening agrees with models.**
- **Any wavelength should yield good quality effective temperatures.**
- **Temperature structure and continuous opacity may be derived with adequate wavelength coverage.**
- **Earliest example was by Hanbury Brown *et al.* (1974) for Sirius, using the intensity interferometer.**
- **First Michelson interferometry observations beyond the first null of the visibility function were reported by Burns *et al.* (1997).**
- **Some other Michelson interferometry examples follow.**

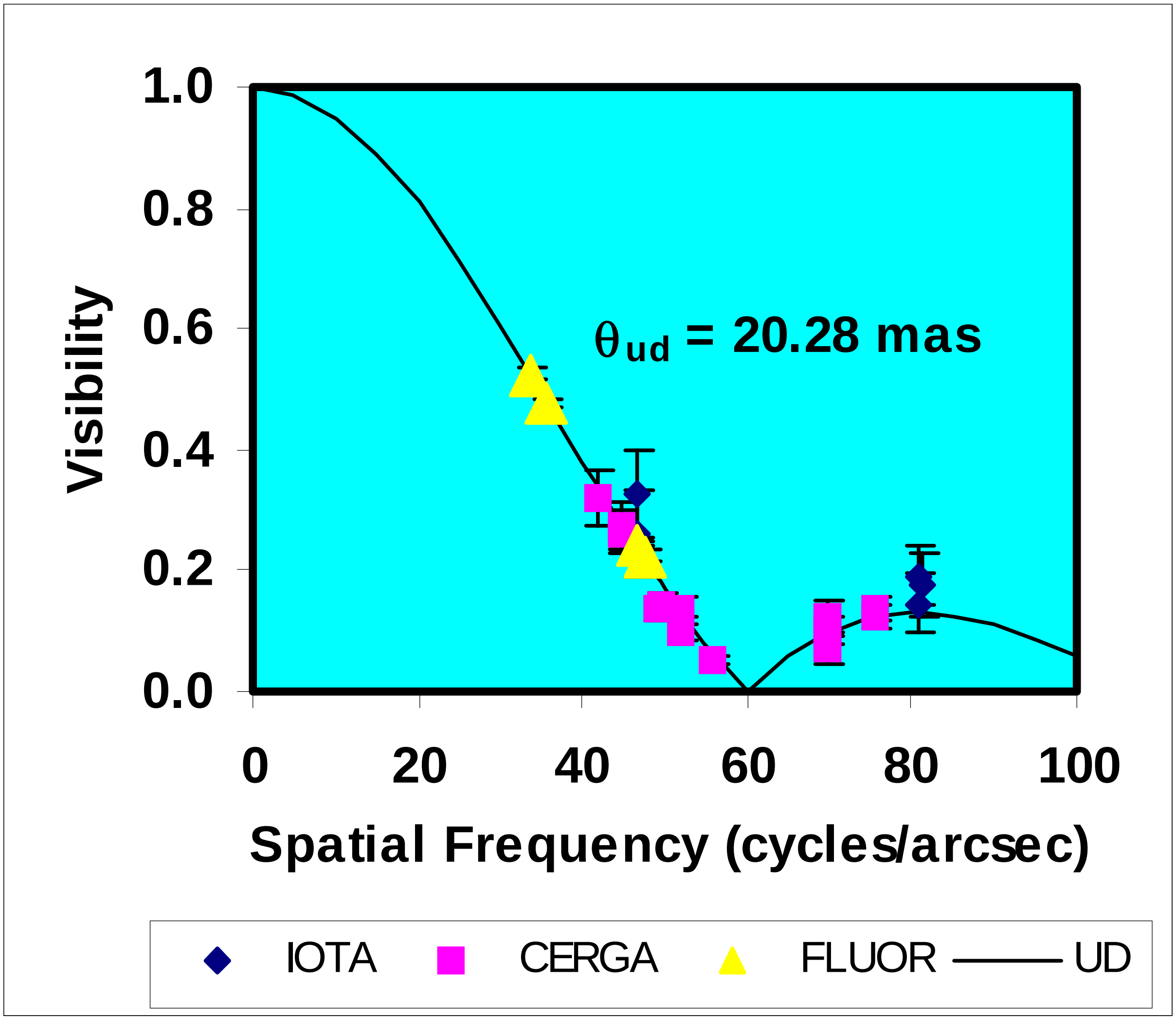
The next two plots are for α Boo, a K2 III star:

The plot below is from Quirrenbach *et al.* (1996) at 550 nm, using the Mark III interferometer.



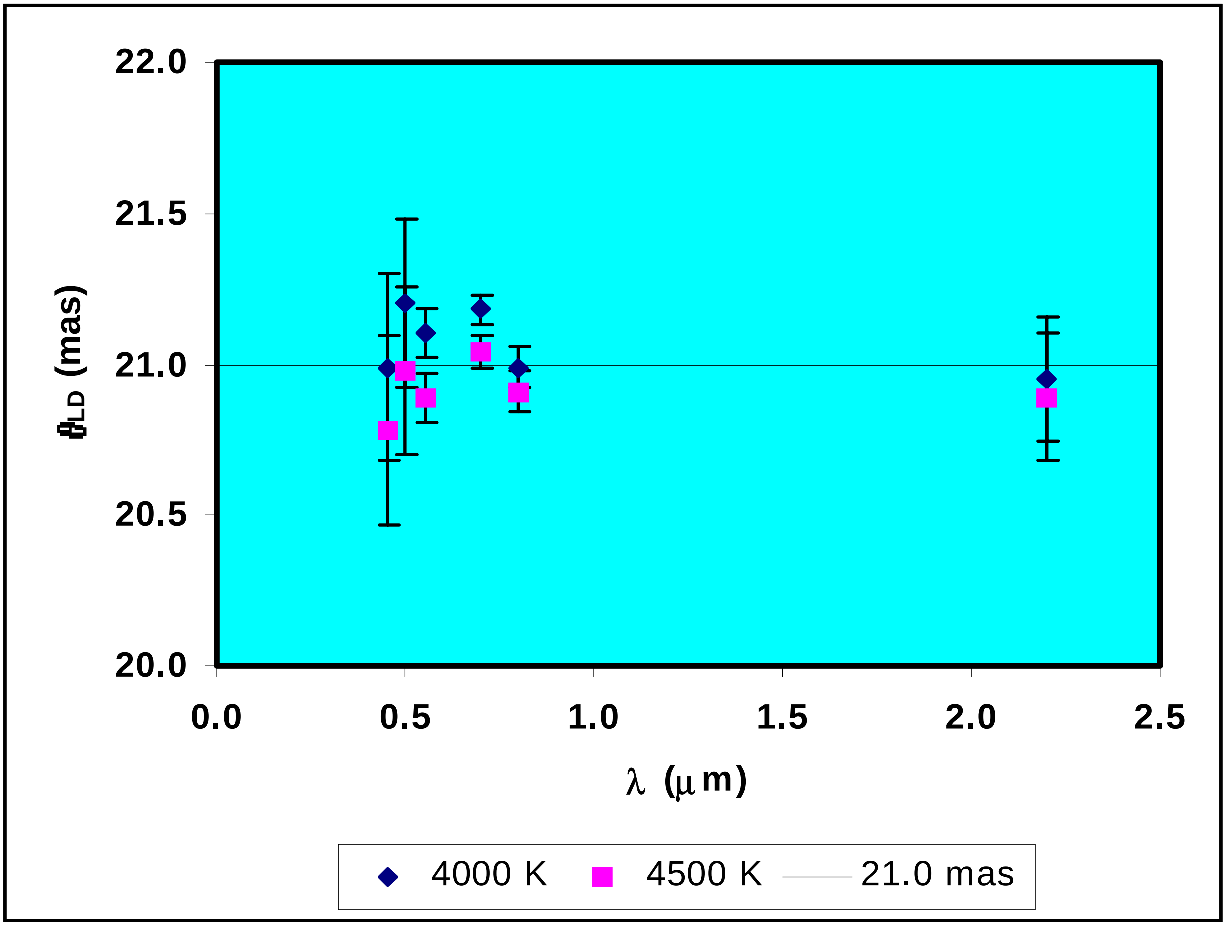
Two attempted fits of uniform disk curves to the data are shown in the figure above.

The plot below is from a variety of observations made at 2.2 µm, using different interferometers.



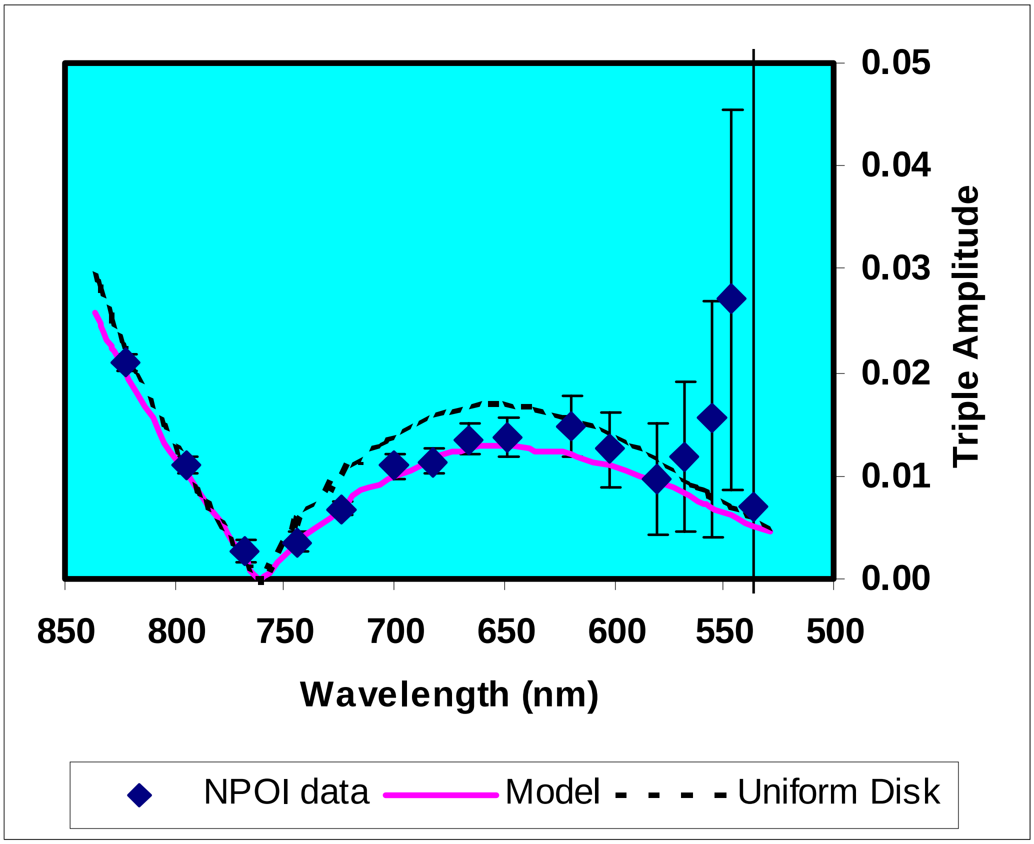
Here, the uniform disk fits reasonably well.

Limb-darkened diameter versus wavelength for α Boo from Quirrenbach et al. (1996):



Note that the limb-darkened diameters are the same at all wavelengths, within the errors. Corrections for two different model temperatures are shown. The best determined diameter is 21.0±0.2 mas.

The following plot is from Hajian *et al.* (1998) for α Cas, a K0 III star:



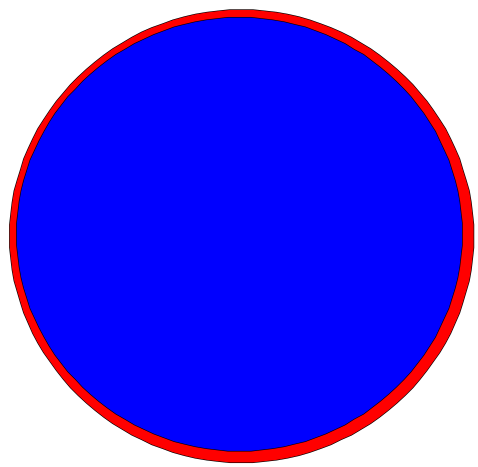
The model above was taken from stellar atmosphere CLV calculations by Kurucz.

What about the cooler stars?

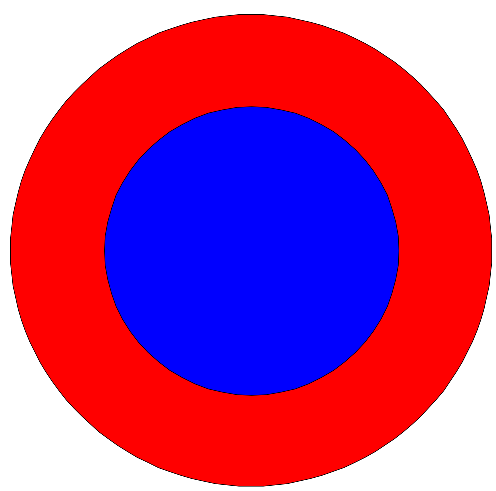
- **Opacities dominated by lines of molecular species.**
- **Atmospheres may be very extended (*e.g*. the Mira variables & supergiants).**
- **Often don't have a sharp 'edge' to the star.**
- **Radius may be very strong function of wavelength, owing to two factors:**
  - **Limb darkening effects**
  - **Significant change of physical depth of the principal radiating layers.**

Pictorial differences

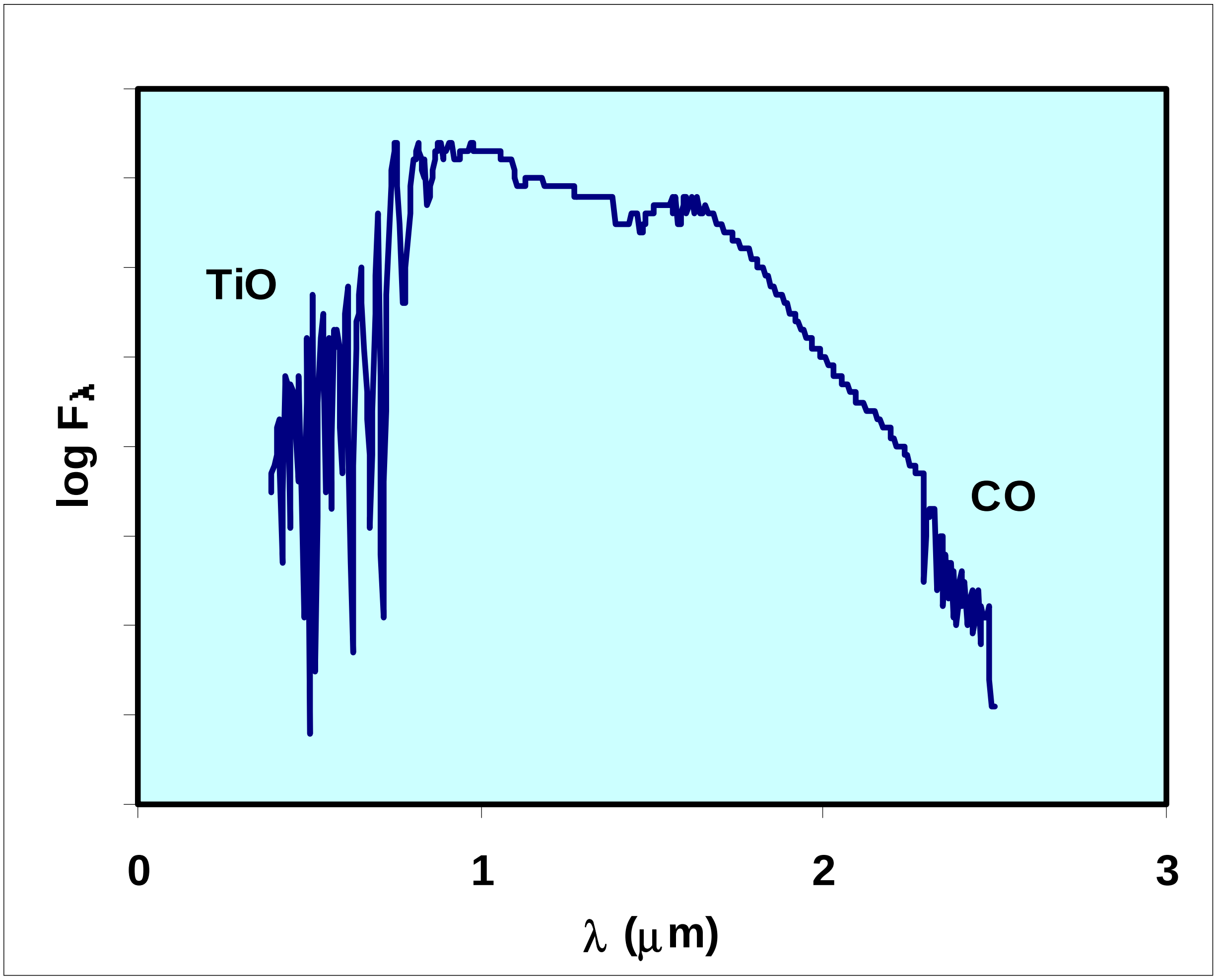
- Warmer stars with compact atmospheres:



- Cooler stars with extended atmospheres:



Computed spectrum for a T = 3500 K, L = 500 L⊙ star, taken from Scholz & Takeda (1987):



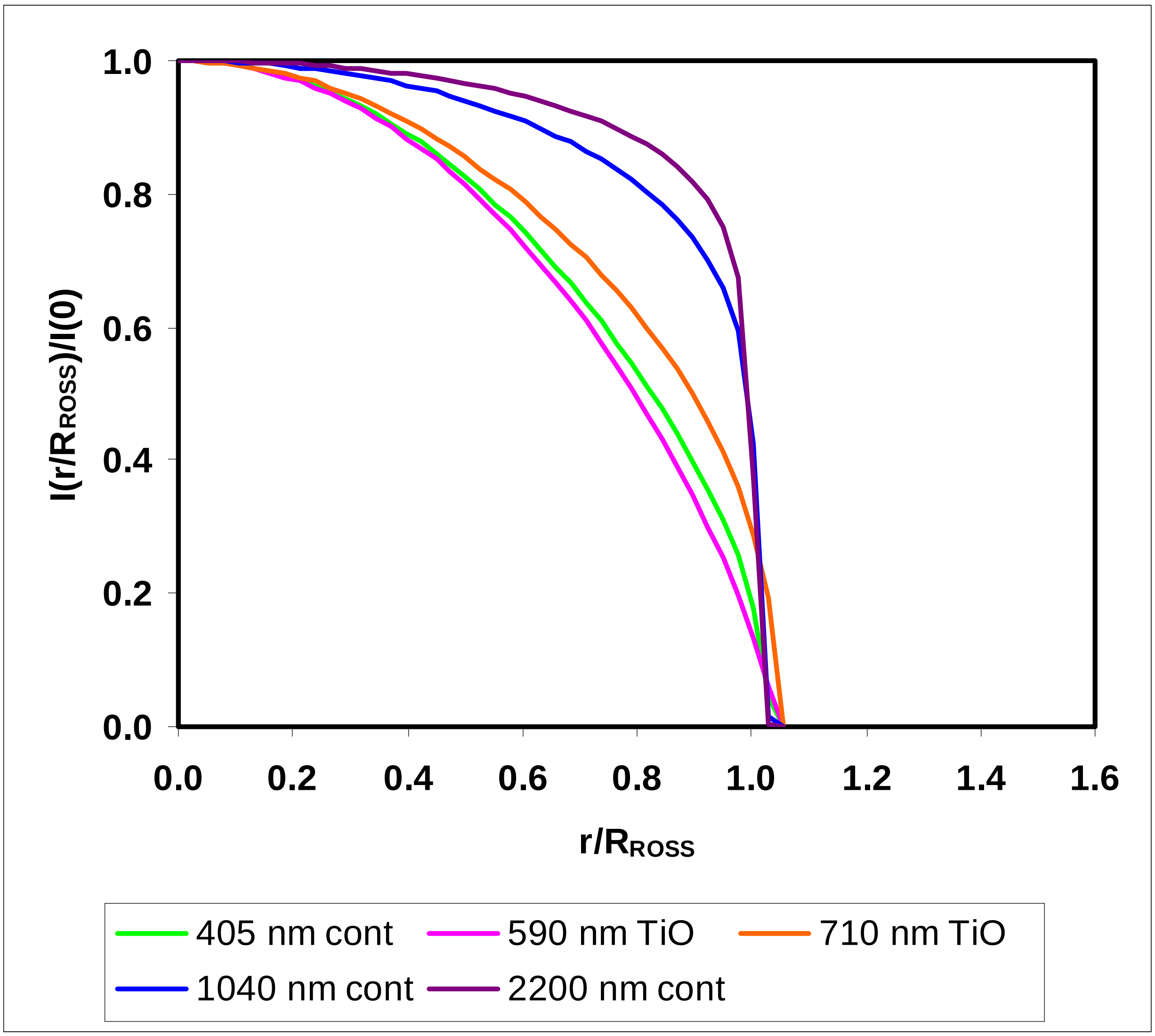
Where is the stellar surface?

- **Introduce new concept — the *Rosseland mean radius*, R_{ROSS} — a fictitious surface:**
  - **Rosseland mean opacity is wavelength averaged opacity.**

$$\frac{1}{k_R} = \frac{\int_0^\infty \frac{1}{k_\lambda} \frac{dB_\lambda(T)}{dT} d\lambda}{\int_0^\infty \frac{dB_\lambda(T)}{dT} d\lambda}$$

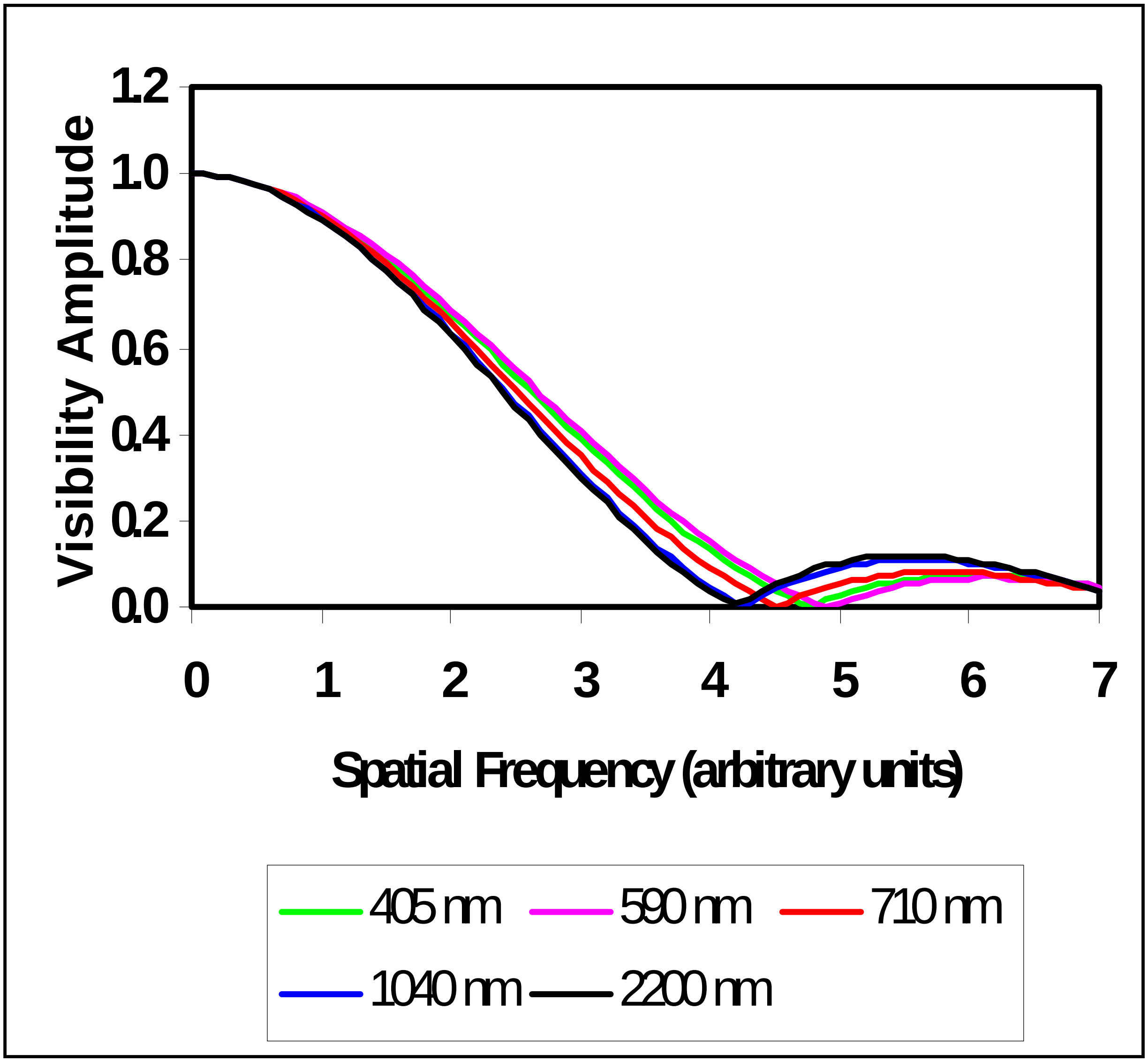
  - **R_{ROSS} defined as physical depth where $\tau_R = 1$.**
  - **Used in effective temperature calculations.**
- **Must relate observed visibilities to this surface through models.**
- **Visibilities may not have same character as visibilities for warmer stars.**

Model CLV for the T = 3500 K, L = 500 L⊙ model, taken from Hofmann & Scholz (1998):

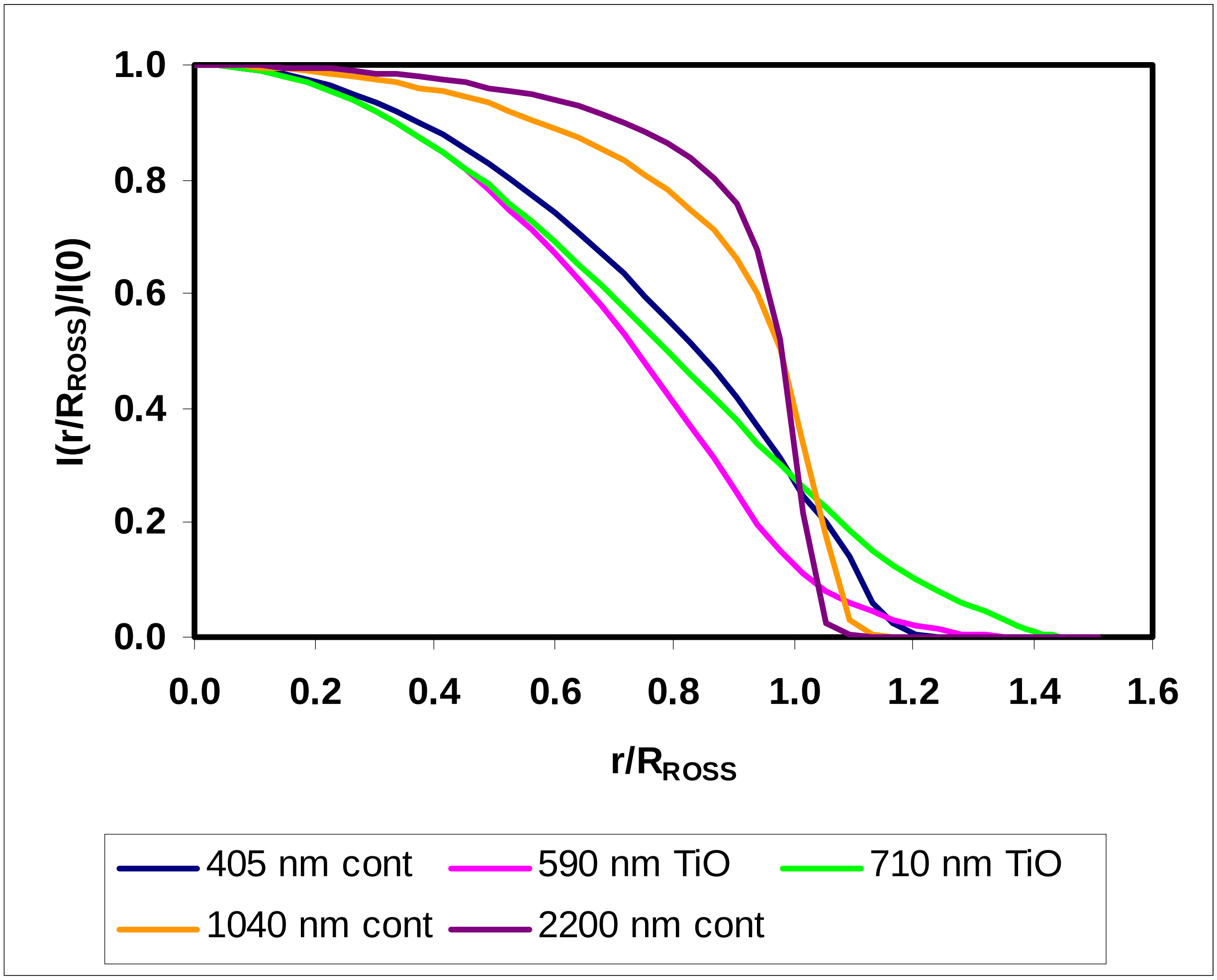


Note the fuzzy edge of the star, as seen in different wavelengths.

Visibility functions for the same model:

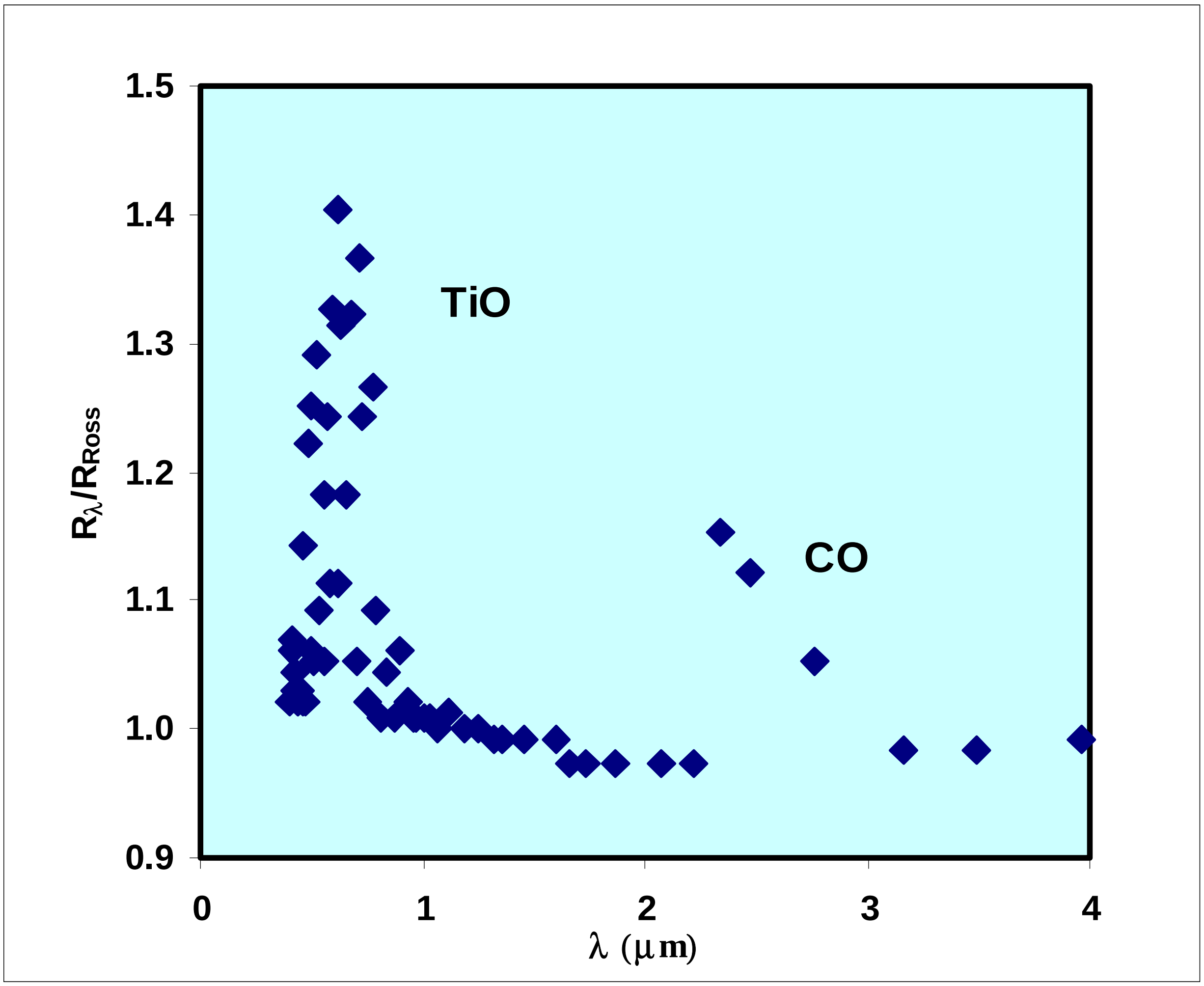


Model CLV for the T = 3500 K, L = 10,000 L⊙ model, taken from Hofmann & Scholz (1998):

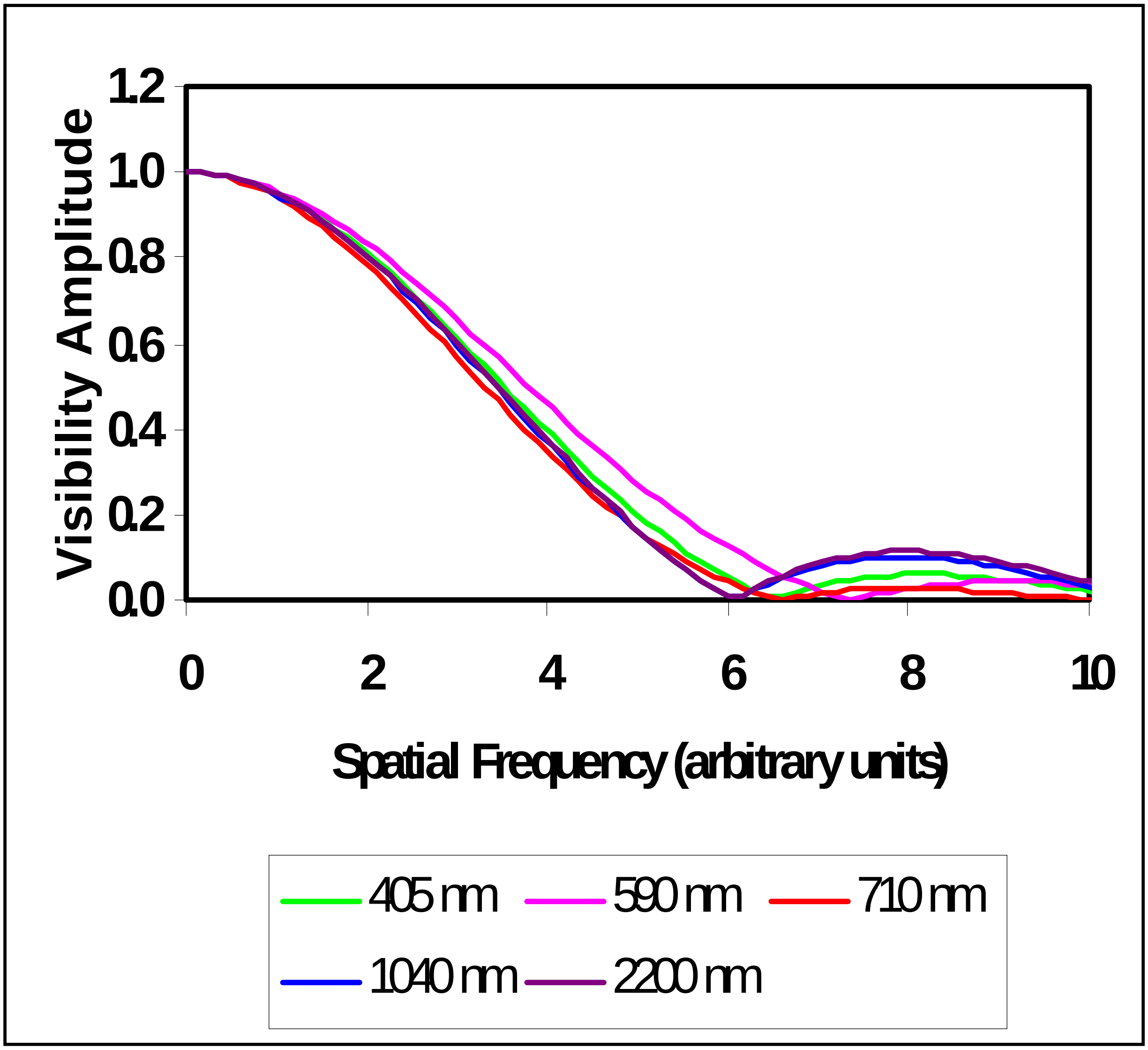


Note *VERY* fuzzy edge to the star for this model.

Change of radius for the same model, taken from Scholz (1985):



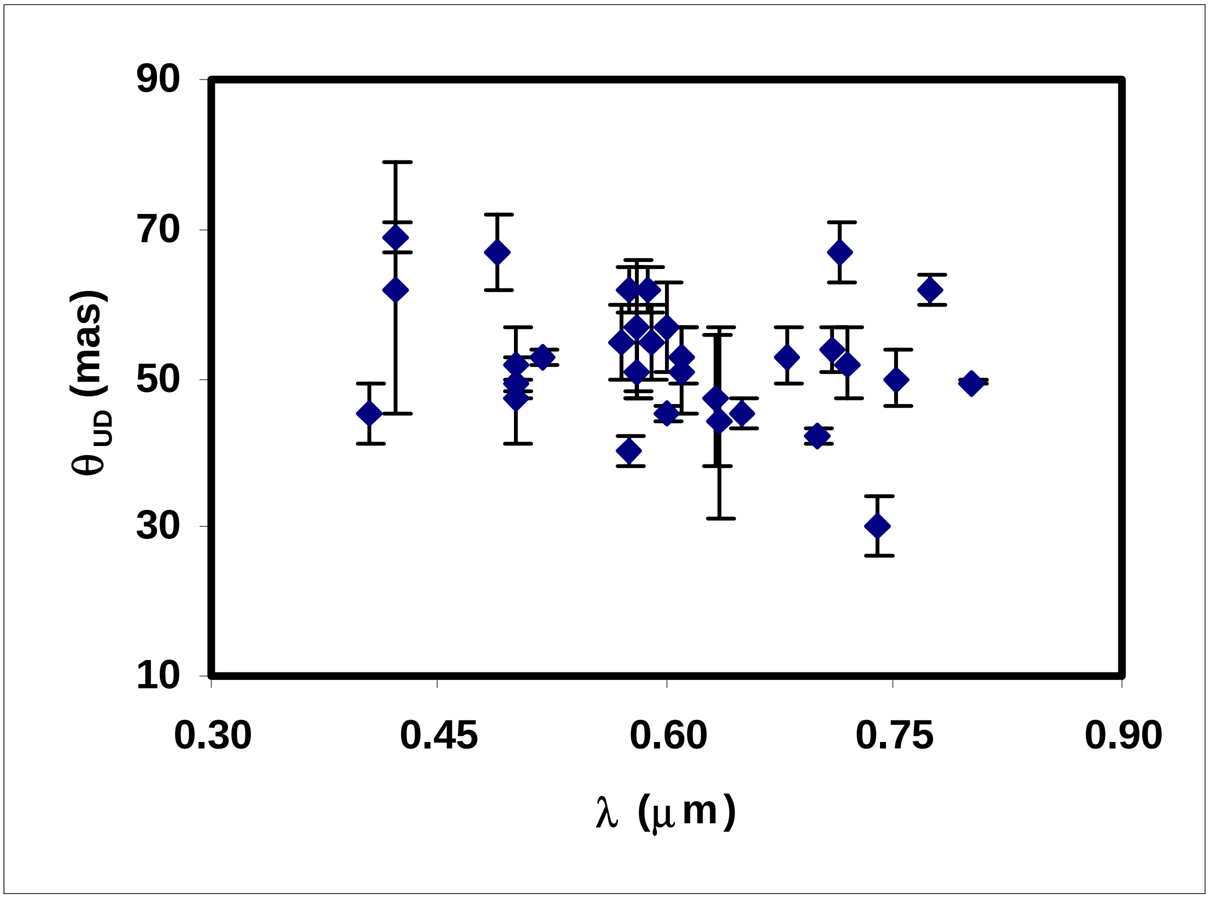
Visibility functions for the same model:



Examples of observations of α Ori (M2Iab):

Observations by Michelson & Pease (1921), Weiner *et al.* (2000) and Gilliland & Dupree (1996) as well as others summarized in White (1980) and Weiner *et al.*

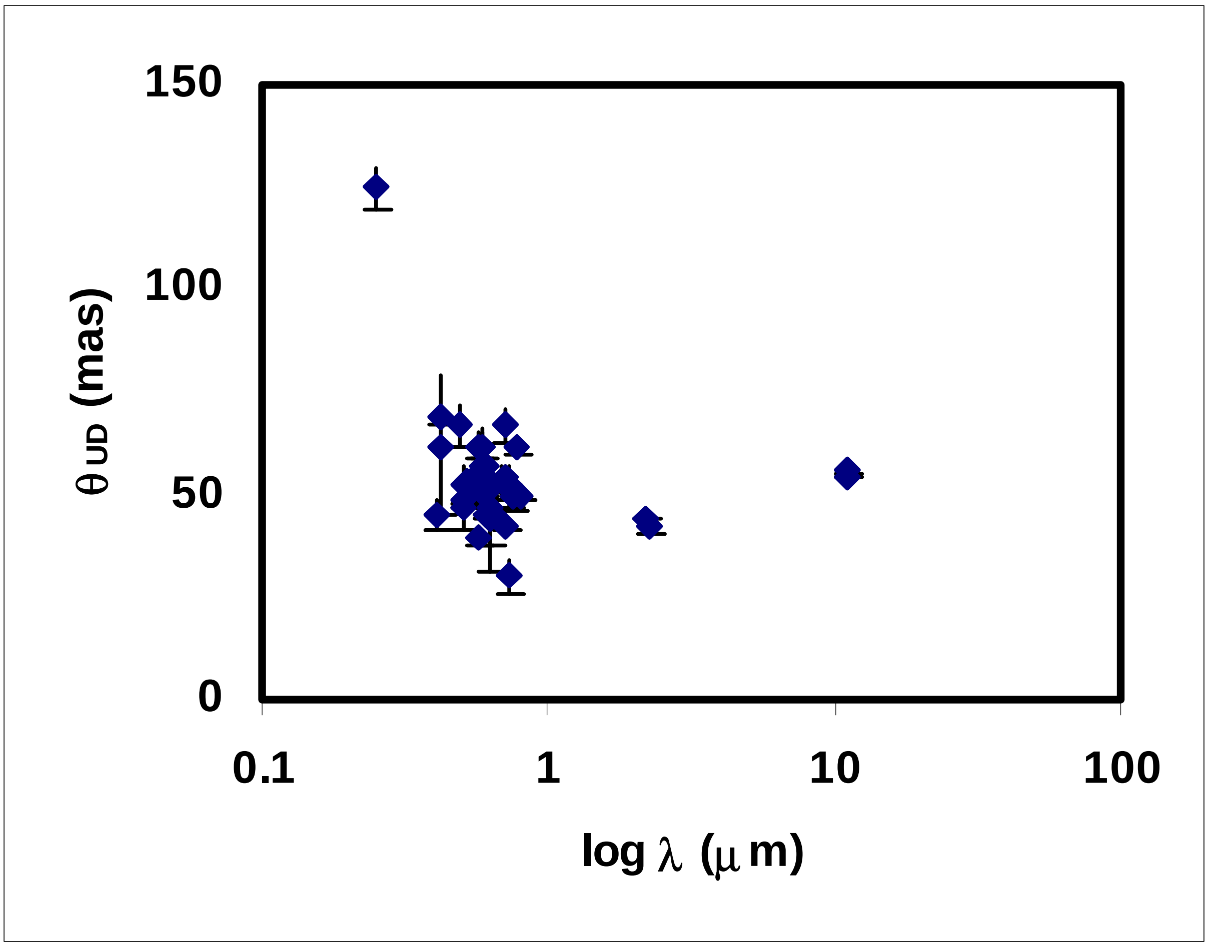
The following is a plot of the uniform disk diameters for α Ori that are summarized in White (1980):



Notes:

- **Much of the scatter may be due to presence of TiO bands.**
- **Some scatter may result from time variability (White 1980).**
- **There is a suggestion of a decrease of diameter with increasing wavelength. Tsuji (1978) explained this by the addition of a circumstellar (scattering) dust shell to the photosphere. The effects of such shells is discussed by Scholz (2000).**
- **The presence of hot spots may complicate the interpretation of the diameter as a function of wavelength (Buscher *et al.* 1990, Wilson *et al.* 1992).**

In the following plot, the previous data plus data from Weiner *et al.* (2000) and Gilliland & Dupree (1996) are shown for α Ori. These extend from the UV to the IR.



Notes:

- **The diameter appears to decrease from the UV into the red and then increase again toward the mid-IR.**
- **The larger diameter in the UV has been explained by Gilliland & Dupree (1996) as a measurement of the chromosphere rather than the photosphere.**
- **The slight increase in diameter from the near to the mid IR may be a contribution from the emission of the same dust postulated by Tsuji (1978). This needs to be tested.**

Summary for cool, extended stars:

- **Variation of angular diameter with wavelength offers wealth of data against which to test spherical, extended model atmospheres (Scholz & Takeda 1987, Hofmann & Scholz 1998).**

- **Important sources of opacity may be characterized with the right kinds of observations (Jacob *et al.* 2000).**

- **Interpretation of visibility curves may be complicated by the existence of circumstellar shells or surface features.**